Title: Future Science: How fuel efficient can cars possibly be?

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Module Summary: Any discussion of sustainability must include an examination of energy consumption by automobiles. The transportation sector (about two-thirds of which is car transportation) accounts for 28.7%[1] of the total US energy consumption, a figure that hasn't changed much for several decades. Internal combustion engines are inherently inefficient, and only about 25% of the energy consumed actually powers movement of the car, so this module will concentrate on these vehicles, with brief mention of alternatives such as electric and hybrid cars.

In this module we will model the energy dynamics of a car, and consider various strategies for reducing the energy necessary to power automobile transport, noting the limitations of those strategies.

Primary Reference: *Sustainable Energy Without the Hot Air* by David McKay

Target Audience: Advanced high school students or college students taking Quantitative Literacy/Reasoning, Precalculus, Environmental Science, Public Policy, or Introductory Physics.

Prerequisites: Comfort with algebraic manipulation of variables, literal equations and units of measurement

Vocabulary:

- **Model**: a theoretical description of a process or event that contributes understanding to how the system works. Models are often somewhat simplified in order to make the most important principles clear, so it is important to know what assumptions underlie the model's construction.
- **Odometer**: the display area on the car's dashboard that shows how many miles the vehicle has traveled. Some cars also calculate trip miles or other subsets of the total.
- **Mileage**: the number of miles traveled per gallon of gasoline.
- **Energy**: often used interchangeably with "fuel" in common usage, energy is more correctly understood as a fundamental driver of physical or chemical change in a system, or the capacity to do work. It can't be created or destroyed, but it can change form, and every time it does (transforms), it becomes less useful for doing work.
- **Fuel**: any material that can release stored energy under specific conditions. For this module, gasoline will be the fuel under discussion, as it releases energy when burned in an internal combustion engine.
- **Power**: energy flow, or how much energy is utilized per unit of time. The watt, for example, is 1 joule of energy used per second. (See Table 1 for more energy and power units.)
- **Literal equations:** equations consisting of variables and mathematical symbols. The variables represent known quantities that change based on a given situation. Often mathematicians and scientists will manipulate literal equations as a time saving tool. An example of a literal equation is the familiar formula for the volume of a rectangular prism (a box) $V = lwh$.

Mathematical Fields: Precalculus, Algebra, Unit Conversions

[1] https://flowcharts.llnl.gov/content/assets/images/charts/Energy/Energy_2017_United-States.png

Goals and Objectives:

Students will:

- Use elementary mathematics to develop a mathematical model for the energy expenditure of a car.
- Use this model to infer physical limitations of cars and answer questions pertaining to environmental policy.
- Apply the concept of energy conservation to a real world setting.

Preparation: Students should (or if they don't drive, acquire similar data from a friend or family member):

1. Two weeks prior to beginning this module, fill the car's tank with gasoline and record the odometer reading.
2. Every day for the two weeks, record the odometer reading at the same time each day.
3. At the end of the two weeks, fill the tank with gasoline again, and record both the odometer reading and the amount of gasoline purchased. Additionally, be certain to record the odometer reading and amount of gasoline purchased any time gasoline is added to the car in this two week period. This data should be recorded in a table.
4. Look up the mass of the car, noting its make and model. If you find the weight in pounds, convert it to kilograms.

The instructor should:

1. Prepare a shared spreadsheet for student data input to be used in Part 1 of the model. Be sure to grant editing rights to your students.

Helpful formulas and facts:

$$Kinetic\ Energy = \frac{1}{2}mv^2$$

$d = vt$, that is $distance = velocity\ \times time$, where *v is the average velocity*

$$1\ megaJoule = 1{,}000{,}000\ joules$$

$$1\ kilowatt = 1{,}000\ watts$$

THE MODULE: PART 1 - modeling car mass and mileage

We will begin by examining the rate of fuel consumption of the cars we drive. Using the data you have collected on the amount of gasoline purchased and your odometer readings, compute your average “mileage” over this time period. Your answer should be in miles per gallon.

Discussion 1: Choose a partner and discuss the process you each used to calculate your car’s average “mileage” over the two week period. Did you use the same process? If not, was one method better than another? Why? Recalculate your “mileage”, if you now have a better process to use.

Activity 1: Enter the mass of your vehicle (in kilograms) and your estimated car “mileage” into a shared spreadsheet provided by your instructor, forming a data set containing the information from all students in your class. (The mass of a particular vehicle can usually be found on the internet - check the manufacturer’s website first.)

Which parameter, mass or “mileage”, should be your independent variable? Copy the data from the shared spreadsheet into your own spreadsheet. Create a scatter plot of this data. Be certain to label your graph well. What might you be able to conclude from the data shown in the graph? Add a trendline to the graph. Does your trendline model the data well?

Write at least one paragraph containing:

- your interpretation of the information conveyed by the graph,
- a discussion of the appropriateness of your trendline, and
- a discussion of factors that may cause the trendline to not be a good model of the data.

Be prepared to provide your instructor with your scatter plot with trendline and the paragraph you have written above.

THE MODULE: PART 2 - how much energy does a car use?

Say you are on a government committee to map out the future of the automobile industry. As our reliance on fossil fuels increases, it is all the more important that we understand how technological innovations can reduce our dependence on these depleting resources. So you are faced with some key questions: what should the fuel consumption of a car look like in the future? How can current automobile designs be improved to increase energy efficiency? And how far can we take this? Is there a limit to the most efficient possible car?

These are science questions, and to answer them we need to develop a mathematical model for how a car utilizes the energy available in its fuel. And in answering these questions, we can begin to address further policy questions that go along with it. What should we change to save fuel?

In order to discuss energy consumption and sustainability, we need to first decide on a way to quantify energy and power.

Table 1 below summarizes different ways of describing energy and power:

Unit	**Equivalents**	**Notes**
Joule	(Kilogram * Meter^2)/sec^2	A quantity of energy
BTU, British Thermal Unit	0.293071 Watt-hours	A quantity of energy
Watt	Joule/sec -or- Newton*meter -or- Kg Meter^2/sec^3	A measure of power (flow of energy)
Kilowatt	Kilojoule/sec -or- 1.34 horsepower	A measure of power, 1000 times a watt (flow of energy)
Kilowatt-hour	3.6 megaJoules	A quantity of energy

The SI unit for energy is the joule (J), which is the amount of energy it takes to apply a force of one newton over a distance of one meter - not the most intuitive measure! To give you a sense of this unit, imagine raising a 1 kilogram mass (2.2 lbs) by 10 centimeters (approximately 4 inches) - you will have expended about a joule of energy in the process.

A joule is a small amount of energy when compared to our daily energy requirements. For example, in 2003, Americans used 896,930,000 joules of energy per person per day (Source: World Resources Institute).

Power is defined as the rate at which energy is used. The watt (W) is the SI unit for power.

$$\boldsymbol{power = \frac{energy\ used}{time}}$$

So one watt equals one joule per second. If your light bulb is rated 40W, that means whenever it is switched on, it's consuming 40 joules of electrical energy in a second.

A useful analogy for the relation between energy and power is that of water volume and the flow rate of water. A drinking fountain may flow at one liter per minute, in which case you'd have to run it for a full minute to fill your one liter water bottle. On the other hand, you could fill it in six seconds with a hose that flows at 10 liters per minute. In both cases, the volume of water delivered is equal to the flow rate multiplied by the time.

$$\boldsymbol{volume = flow\,rate\ \times time}$$

$$\boldsymbol{energy = power \times time}$$

Just as water volume is a quantity, but flow rate is a water quantity delivered over time; so is energy a quantity, but power is an energy quantity delivered over time. When a 1000W toaster is switched on, it consumes energy at a rate of 1000 joules each second it is on. That's a much more powerful consumption rate than our light bulb!

As you can see, joules and watts are not human-sized units. A more convenient unit of energy that is commonly used to measure daily usages is the kilowatt-hour (kWh), which is an energy flow (kilowatt) multiplied by time (1 hour) to get back to a specific quantity of energy (the number of kilowatts delivered in a single hour). If you look at your utility bill, you will find your monthly usage listed in kilowatt-hours - the standard measure of how much electricity you're using.

When dealing with car engines, however, we don't usually use kWh as a unit of energy (although it is possible - 1 kilowatt hour equals 3.6 megaJoules of energy). Kilojoules will be more direct and useful. First let's look at the combustion of gasoline itself. Gasoline is a fuel, which means it holds energy for future release. Burning it in an internal combustion engine releases a specific quantity of energy per amount burned. (Notice this is not per time, but rather per volume burned). Let's assume you travel 40 miles per day--the approximate average for "traditional" college-aged people - and that the mileage is the one you measured in your own car in miles/gallon. The amount of energy your car consumes each day is then equal to:

$$\boldsymbol{Energy = (number\ of\ gallons\ of\ fuel\ consumed\ in\ a\ day) \times (energy\ per\ gallon\ of\ fuel)}$$

$$\boldsymbol{Energy = \left(\frac{40\ miles}{mileage}\right) \times (energy\ per\ gallon\ of\ gasoline)}$$

There is a known quantity of potential energy in each gallon of gasoline (a mixture of several compounds). For this module we will use 131,760 $\frac{kJ}{gallon}$ of gasoline for that potential energy.
If your average mileage is 25 miles/gallon, then you have used

$$\boldsymbol{\frac{40\ miles}{25\ mpg} \times 131{,}760\,\frac{kJ}{gal}}\ \textbf{ of energy on an average day of driving}$$

Regrettably, an internal combustion engine is not very efficient. Notice that a car with mileage of 50 mpg would use only half the energy for the same gasoline and miles traveled. Mileage also varies dramatically with conditions. A car uses more energy going uphill, going faster, and in start and stop driving. Even idling uses energy for zero forward motion. If you have a newer car, you may have a sensor that computes mpg every few seconds and displays it on the dash. Finally, even in relatively high mpg, even going downhill, much of the energy from combustion is wasted as heat.

Activity 2: At what speed do you typically drive in the city? How about on interstate highways? If your primary concern is fuel efficiency, at what speed should you drive if you are visiting your friend who lives in a nearby city? Assume that you encounter traffic lights and stop signs for the first 4 miles and the last 3 miles when you go to visit a friend who lives 40 miles away. Otherwise, your trip is made on an interstate highway.

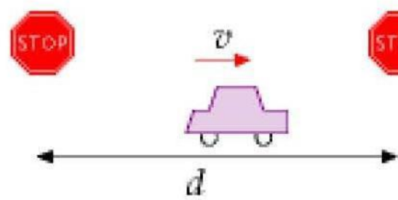
Discuss these questions and write down your answers.

THE MODULE: PART 3 - Where are the inefficiencies?

NOTE: In this part of the module we will be working with literal equations.

We saw above that the average energy consumption of your car per day is on the order of 211 megajoules (or 58.6 kWh) - much of it not going to the movement of the car. In what ways does this energy end up being consumed? We often hear of more "energy-efficient" cars. Energy dissipation depends on the driver and the terrain, but what about the properties of the car itself? Write down some things about the car that you think might be affecting gas mileage and discuss your ideas with a classmate.

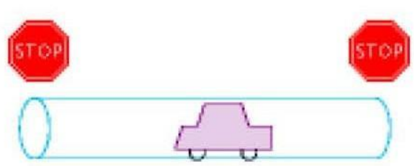
We'll begin by building a model of the energy consumption of a car. Assume that the driver starts from rest, accelerates to a cruising speed of *v* miles per hour, and then maintains this speed, *v*, over a distance of *d* miles. This could be the distance between traffic lights or stop signs.



Any moving car has a certain amount of energy of motion associated with it. This is the minimum amount of energy that it took to bring it to this speed, and also the amount of energy you would need to dissipate in stopping the car.

Question 1: When moving at a speed of *v*, what kind of energy does a car possess due to its motion? How can this be expressed numerically?

So, back to the driver who is cruising at a speed of *v.* The moving car possesses kinetic energy and, after covering a distance *d,* the driver slams on the brakes, bringing the car to a stop. This converts all of the car's kinetic energy into heating up the brakes (and some energy into the screeching noise). In a traditional car, the energy dissipated through braking is wasted. Today's hybrid and electric cars harvest some of this energy and store it in special batteries that serve as fuel for the car in lieu of gasoline, under certain conditions. For the purposes of this module, we are only concerned with traditional gasoline engine cars.



Braking is not the only way for energy-loss to occur. Another way that a moving car consumes energy is by making air swirl around it as it moves. When moving, a car pushes air out of its way, creating a volume of swirling air behind it, which moves at a speed close to *v*. Since this displaced air is moving, it possesses kinetic energy as well.

To summarize, the main ways in which energies are dissipated from the car are by slowing down the car using brakes, and pushing the air out of the way as the car moves forward (air resistance).

Let's take a closer look at these two kinds of energy.

1) **Braking energy.** If the mass of the car is m_c and it moves a distance, ***d*,** at average speed, ***v*,** before coming to a stop, then the rate at which energy dissipates into the brakes is given by

$$\textit{Average Power} = \frac{\textit{kinetic energy of the moving car}}{\textit{time between stops}}$$

Question 2: Express the rate at which energy dissipates into the brakes in terms of the mass of the car, the speed, and the distance between stops (when the instantaneous velocity is 0).

2) **Air Resistance.** This is the energy it takes to bring a stationary volume of air up to a speed *v* (or, if you prefer, the kinetic energy of the displaced air).

Question 3: If the mass of the displaced air that is being accelerated is m_{air}, express the kinetic energy of the displaced air in terms of the mass of the displaced air and its speed.

How do we compute the mass of the moving displaced air? If we know the density of air, we can express mass in terms of volume.

$$\textit{mass} = \textit{density} \times \textit{volume}$$

Question 4: What is the mass of the displaced air swept out in a given period of time, *t*? Express your answer in terms of the cross sectional area, *A,* the speed, *v*, and the time period, *t*. You may assume that the air is moving at the same speed as the car. (We will come back to the possible shape of the cross-sectional area shortly. For now, just let *A* represent the cross-sectional area.)

Question 5: Now that we have an expression for the mass of the displaced air, **derive an expression for the rate at which energy is spent in moving the displaced air** (express your answer in terms of the density of air, ϱ (the Greek letter rho), the cross sectional area of the car, *A*, and the speed of the car, *v*).

Discussion 2: What is a good approximation for the cross-sectional area of the displaced air? How would it change for a more streamlined car?

For the purposes of this module, we will use a simplified depiction of the displaced air as a revolving cylinder.

So, considering the two main ways in which energies are dissipated from the car, we now have a model for the total rate of energy consumed by the car

Power of the car = power going into the brakes + power in pushing air out of the way.

$$\textit{Power of the car} = \frac{1}{2}\frac{m_c v^3}{d} + \frac{1}{2}\varrho A v^3$$

Question 6: Both these rates of energy expenditure scale as v^3. How does a driver who halves her speed change the energy expenditure of the car? If she drives the same distance, how much longer will the journey take? And how will the energy consumption change?

THE MODULE: PART 4 – Where Should We Focus to Improve Efficiency

Our model takes into account two primary ways energy is dissipated while driving, namely braking and air resistance.

The rate of energy spent in braking is

$$\frac{1}{2}\frac{m_c v^3}{d}$$

The rate of energy spent bringing air to the speed of the car is

$$\frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{\varrho}\mathbf{A}\boldsymbol{v}^{\mathbf{3}}$$

If we want to improve our car's efficiency, then a natural starting point is to determine which of the two energy dissipations is bigger.

Activity 3: Determine whether more power goes into the brakes or into pushing the tube of air. Your answer should depend on whether the mass of your car is greater or lesser than another quantity. What is the physical meaning of this other quantity?

To get started, it might help to consider the ratio of the two terms i.e.

$$\frac{\boldsymbol{power\ spent\ in\ braking}}{\boldsymbol{power\ spent\ in\ pushing\ air}}$$

and determine if that ratio is more or less than 1. Form the ratio and simplify it, showing all of your work.

What does your car mass need to be for energy dissipation to be dominated by braking? What does it need to be for energy dissipation to be dominated by air resistance?

Answer: The ratio simplifies to

$$\left(\frac{\frac{m_c}{d}}{(\varrho\ \mathrm{A})}\right)$$

It is bigger than 1 if $m_c > \varrho\ \mathrm{Ad}$. Since Ad is the volume of the tube of air swept out as the car travels *d* feet, *ϱ Ad* is the mass of that same tube. Thus energy dissipation is dominated by braking when the mass of the car is greater than the mass of the tube of displaced air. Otherwise energy dissipation is dominated by the loss due to air resistance (also known as drag).

You should have found that:

if $m_c > m_{air}$ driving is braking dominated and, if $m_c < m_{air}$ driving is drag dominated

In other words, the dominant form of energy loss depends on the type of driving we are doing. In particular, it depends on whether the mass of car is greater or lesser than the mass of the tube of air displaced by the car (starting from the time it accelerates to the moment it brakes to a halt).

Let's focus on the mass of the tube of air, $m_{air} = \varrho\ \mathrm{Ad}$. This depends on a few factors. First, there is *d*, the distance between stops of the car. We'll talk about this in a moment. Then there's the cross sectional area, *A*. In reality, this is composed of two terms

$$effective\ cross\ sectional\ area\ A = drag\ coefficient\ \times cross\ sectional\ area\ of\ the\ car$$

$$(\boldsymbol{A} = \boldsymbol{c_d A_{car}})$$

The drag coefficient is a fraction that has to do with how streamlined your car is. More streamlined shapes have smaller drag coefficients. For example, swimmers can form a stream line with their arms and minimize their drag coefficient when diving into water. Think about the difference between a diver in a streamlined configuration entering the water and producing very little splash versus that same diver doing a "belly flop". A more streamlined car reduces the amount of motion created in the air, thus reducing the kinetic energy of the displaced air.

Question 7: Determine the distance *d* at which both terms are equal. You can also think of this as the distance that separates brake-dominated driving from drag-dominated driving. If you know the mass of your car, you can use that. Otherwise, you can approximate it by 1000 kg. You will need to estimate the cross-sectional area of your car, in order to calculate the volume of the tube of displaced air. A reasonable estimate might be 3 meters squared. Finally, you'll need the drag coefficient, for which you can use ⅓.

Question 8: Using this threshold distance, ***d,*** argue that in "stop-and-go" traffic, such as city driving, braking is the primary source of energy loss, whereas highway driving is dominated by air resistance.

Question 9: How can you improve your car's efficiency in city driving? Consider the braking term of the energy loss formula and describe two ways of improving efficiency. How about highway driving? Use the air drag term to come up with three ways of improving efficiency.

So far we've neglected one significant source of energy waste: engine inefficiency. A typical gasoline engine uses only 25% (or 1/4th) of fuel energy towards powering the car--the rest serves only to heat up the engine. That is, the other 75% of fuel energy is transformed into heat energy. This excess "waste" heat must be sent out from the car through the muffler and the radiator (or air cooling) system. So to get a realistic estimate of the total power of a car, we need to scale our formula by a factor of 4:

$$\boldsymbol{total\ power\ of\ the\ car} = \mathbf{4}\left(\frac{\mathbf{1}}{\mathbf{2}}\frac{\boldsymbol{m_c v^3}}{\boldsymbol{d}} + \frac{\mathbf{1}}{\mathbf{2}}\varrho\, \mathrm{A} v^3\right)$$

Activity 4: We will now plug in plausible numbers to the above formula to determine the amount of power a typical car might expend while moving. Let's first consider highway driving at a speed of 70 miles per hour. You can assume the braking distance is large enough to render the first term insignificant. (70 miles per hour = 110 km/h = 31 m/s) Using ⅓ as our drag coefficient and our reasonable estimate of the cross-sectional area of 3 meters squared, $A = c_d A_{car} = 1m^2$.

If you were to drive at this speed for one hour, how much energy would your car expend (in kWh)? What if you drove at half this speed?

Question 10: Using the model we've built, argue whether the energy savings from a lower mass electric car will be more substantial in city driving or in highway driving. Support your argument using one of the equations derived above.

THE MODULE: PART 5 - Flipping Fuel Efficiency

Thus far, this sustainability module has followed the convention in the United States by looking at fuel efficiency as distance divided by the amount of fuel consumed, miles per gallon. Under this way of quantifying fuel efficiency, answer the following.

Question 11: A car is driven for 90 miles and consumes 5 gallons of gasoline. Calculate the car's mileage.

Question 12: Another car is driven 90 miles and consumes only 4 gallons of gasoline. Calculate the car's mileage.

Question 13: A third car is driven 90 miles and consumes just 3 gallons of gasoline. Calculate this car's mileage.

Question 14: A fourth car is driven 90 miles and consumes a mere 2 gallons of gasoline. Calculate this car's mileage.

Discussion 3: Discuss with a classmate: Is there a bigger gain in fuel efficiency by (a) driving the second car instead of the first, OR (b) by driving the fourth car instead of the third?

Question 15: "A town maintains a fleet of vehicles for town employee use. It has two types of vehicles. Type A gets 15 miles per gallon. Type B gets 34 miles per gallon. The town has 100 Type A vehicles and 100 Type B vehicles. Each car in the fleet is driven 10,000 miles per year. The town wants to replace these vehicles with corresponding hybrid models in order to reduce gas consumption of the fleet and thereby reduce harmful environmental consequences.

Should they (1) replace the 100 vehicles that get 15 mpg with vehicles that get 19 mpg, or (2) replace the 100 vehicles that get 34 mpg with vehicles that get 44 mpg?" (This problem is shamelessly copied from a survey by Richard Larrick and Jack Soll of Duke University.)

Provide a justification for your choice.

Time to Flip

Now, let's turn this world upside down. Instead of looking at distance divided by the amount of fuel consumed, let's examine these same questions by calculating fuel efficiency by the number of gallons consumed divided by distance (gallons/mile or gpm). The scientists at Popular Mechanics and other forward thinking folk feel we should always consider fuel efficiency from this "upside down view".

Activity 5: We will now rework the previous portion of PART 5 of this module:

- Recalculate the fuel efficiencies of the four cars (from questions 11 - 14) by calculating the gpm for each car.
- Now, looking at these calculations revisit Discussion 3.
- Answer Question 15 from the point of view of gpm fuel efficiency. Justify your answer.
- Finally, write a paragraph identifying your preferred method of measuring fuel efficiency (mpg or gpm) and provide your rationale. That is, persuade the reader to agree with your choice.

The contents of this teaching module are based on *Chapter 3* and *Technical Chapter A* of David McKay's excellent book *Sustainable Energy Without the Hot Air*, available free online at http://www.withouthotair.com