Title: Carbon Footprint: A Study of Unit and Dimensions

Authors: Laura Foster, Rikki Wagstrom, Jean McGivney-Burelle

Module Summary: In this module, we integrate the context of carbon emission and human consumption into an introductory lesson on units and estimation. Background information is provided to familiarize students with the science of carbon emissions as well as greenhouse gas effects on mean global temperatures.

Informal Description:

Target Audience: General education mathematics or higher

Prerequisite Math: Fractions, scientific notation, metric system

Math Fields:

Technology: Calculator

Applications: Environmental Awareness, Climate Science

Goals:

As a result of this module students will

- Possess a deeper understanding of the science of carbon emissions
- Understand how choices they make (e.g., walking vs. riding a bike vs. driving, using different types of light bulbs) impact environment
- Understand the need to make and clearly state assumptions made for solving carbon emission problems
- Use data to answer questions and make predictions about carbon footprints
- Be able to work with units and unit conversion

Resources

- Websites:
  - http://www.esrl.noaa.gov/gmd/ccgg/trends/ (website with Mauna Loa CO_2 data)
  - http://www.epa.gov/cleanenergy/energy-and-you/how-clean.html (website that provides information on CO_2 production by zipcode)
  - http://www.digitaldutch.com/unitconverter/ (unit converter)
- Textbooks: general chemistry text
- Unit conversion chart

Module

This module is appropriate for mathematics classes at the general education level or higher. The time required to complete the module in its entirety will depend on the student's level of familiarity with the material in the module. Based on preliminary testing of the module in general education mathematics courses, two 50-minute class meetings are recommended to complete the entire module. For more advanced courses (post calculus), the module might be completed within a single class meeting. Having said this, the module organization is flexible. Instructors teaching general education courses may choose to incorporate only the first part of the module if time is limited.

This may be thought of as a two-part module. The first part introduces students to the ideas of dimension, units, and unit conversion. Building from this introduction, the second part applies the ideas of unit conversion to the topic of plausible estimation, including a small-group activity where students estimate carbon emissions associated with transportation. Accompanying the module is a reading assignment to introduce students to the idea of using unit conversions to make plausible estimations of carbon emissions from energy usage. Additionally, there are two homework assignments (I and II). Assignment I includes follow-up questions from the reading assignment as well as some introductory-level exercises to assess student's understanding of unit conversion. Assignment II enables students to practice making assumptions and plausible estimations in diverse contexts.

For General Education Courses We suggest that one class meeting be spent on each of the parts of the module (two classes total). For instructors requiring a shorter module, part two of the module can be omitted. We suggest that students complete the reading assignment before the first class meeting (particularly if only one class meeting is allotted for the module), but some instructors may choose to assign the reading after they've had an opportunity to discuss unit conversion with their students. Homework assignment I may be assigned after the first class meeting, while assignment II may be assigned after the second meeting.

For Advanced Courses For students that are already familiar with unit conversion or are mathematically advanced, part one of this module can either be skipped entirely or quickly reviewed, enabling instructors to devote the bulk of class time to part two. Depending on the level of the students, instructors may choose to assign both the reading assignment and homework assignment I as a pre-class assignment, while assignment II would be an appropriate post-class assignment.

To meet the goals of this module, we have included both student and instructor materials. Student handouts are found on pages 3-13. Instructor materials are found on pages 14-29. The instructor materials include notes to facilitate teaching unit conversions and estimation during class, as well as detailed solution keys for the two homework assignments and in-class activity.

Reading Assignment

Introduction

In the fall of 2007, Al Gore and the Intergovernmental Panel on Climate Change (IPCC) were jointly awarded the Nobel Peace Prize for their work to disseminate information about the causes of, the predicted effects of, and measures needed to counteract global climate change. The IPCC is a United Nations organization of international scholars whose purpose is to provide assessment of the causes and risks of anthropogenic (human-caused) climate change. Their 2007 Synthesis Report summarizes the causes and predicted outcomes of climate change on society and ecosystems. The report details the growing consensus among scientists that data showing increases in ocean temperature and sea level and a decrease in snow cover provide clear indication of global warming, as demonstrated in the data presented in Figure 1. The conclusion that our Earth is warming is supported by much more numerical and scientifically measured data. However, a pictorial example may prove to be more convincing and demonstrative. The pictures in Figure 2 show the decrease in the size of the Boulder Glacier in Glacier National Park between 1932 and 1988. This is not only a great visual example but is also a very important example demonstrating the impact of global warming on human existence. Communities living near such glaciers depend on these icy giants as sources of fresh water. As the glaciers melt permanently, these sources of fresh water disappear, threatening the survival of these communities.

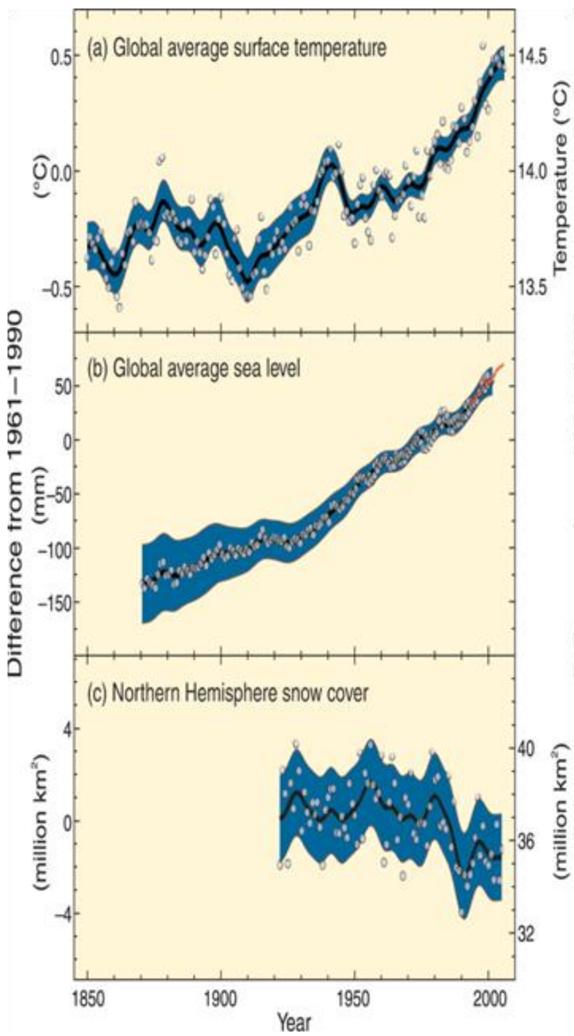

Figure 1. Data demonstrating the difference in (a) Earth's average surface temperature, (b) the average global sea level, and (c) the amount of snow cover in the Northern Hemisphere. The differences are relative to averages for the period from 1961 to 1990. Source: IPCC AR4.

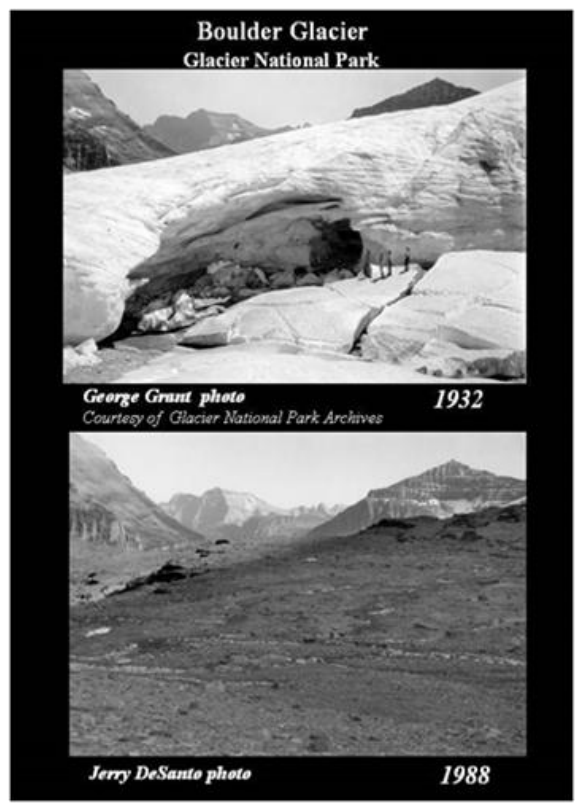

Figure 2. Photos depict Boulder Glacier in Glacier National Park in 1932 and 1988. Source: Glacier National Park Archives.

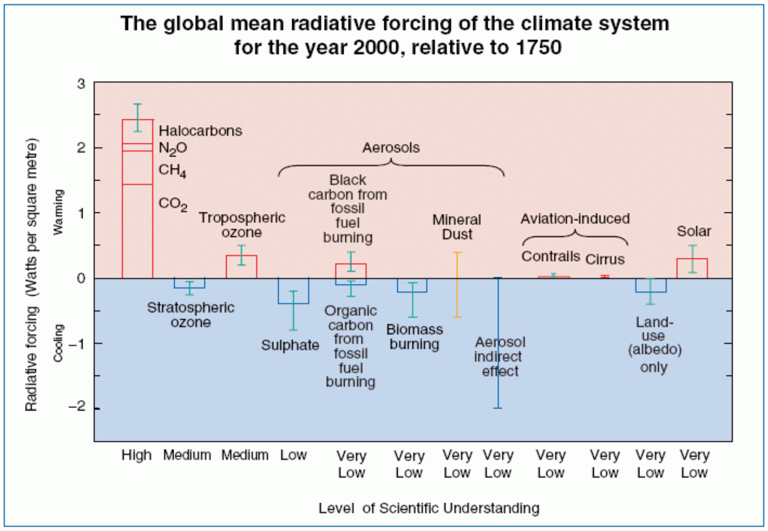

Figure 3. Difference in mean radiative forcing for different constituents that contribute to changes in global temperature plotted as a function of the level of our scientific understanding for each constituent. Note that the first column on the right represents global warming due to greenhouse gas emissions.

To better understand the link between human activity and climate, consider the diagram in Figure 3. This graph plots the mean radiative forcing, which is simply the average global warming potential, of different constituents on an x-axis that indicates our level of scientific knowledge about the constituent. The y-axis can be read as the difference in the radiative forcing value for the specific constituent between the years 2000 and 1750. Any value above the x-axis indicates that the constituent contributes to global warming; any value below the x-axis indicates that the constituent contributes to global cooling. The contributions from constituents that cause warming greatly outweigh the contributions that would cause cooling; therefore, we see a warming trend. Understanding all the constituents and their effects is well beyond the scope of this lesson and would take an in-depth exploration into atmospheric chemistry and physics to begin to understand. Note that many of the constituents are labeled "very low" for their level of scientific understanding. This means that even climate scientists do not fully understand the full effect on global temperature that these constituents have. These are areas of on-going scientific research.

An initial question may arise from the title. What is significant about the year 1750? You may remember from your history classes that the Industrial Revolution began in the mid-eighteenth century. This boom in technological advancement marks the beginning of widespread use of burning fossil fuels to produce energy. Since this time, the resulting gas and particle by-products of burning fossil fuels have released into the atmosphere, thus altering the composition of the air. The constituents represented by columns 1, 3, 4, and 5 of Figure 3 are all attributed to the fossil fuel combustion.

In this module, we will focus our attention on the contributor where there is a high level of scientific understanding about change in radiative forcing. The first column on the x-axis represents the change in radiative forcing due to the increase of greenhouse gases in the atmosphere. The different segments of the bar represent the greenhouse gases carbon dioxide (CO_2), methane (CH_4), nitrous oxide (N_2O), and halocarbons (primarily man-made molecules used as refrigerants and propellants such as Freon-12 (CF_2Cl_2)). A greenhouse gas works like the glass in a greenhouse: it allows the sunlight to enter the system but does not allow the heat to escape. While many of these gases occur naturally, their levels have risen dramatically since the 18th century due to human activity. Their impact on global temperature can be understood by explaining some atmospheric chemistry and basic molecular properties of gases.

Science behind greenhouse gasses

Earth's atmosphere is primarily nitrogen (N_2: 78%) and oxygen (O_2: 21%) and argon (Ar: 1%) with trace levels of many other gases, including the ones mentioned above. For instance, CO_2 currently accounts for about 0.0392% of the atmosphere. It is important to note that water is also a greenhouse gas; however, the amount of water in the atmosphere has not changed significantly since the Industrial

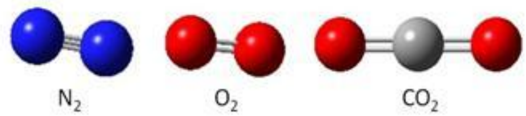

Figure 4. Figure 4. Molecular structure of nitrogen (N_2), oxygen (O_2), and carbon dioxide (CO_2).

Revolution. The presence of nitrogen, oxygen, and argon do not significantly affect the temperature of our atmosphere, while CO_2 does. Or put differently N_2, O_2, and Ar are not greenhouse gases, while CO_2 is. The molecular structures of the different gases help explain why this is true. Figure 4 shows the molecular structures for N_2, O_2, and CO_2. N_2 and O_2 have hemolytic bonds, meaning the bonds join to atoms of the same species: CO_2 has heterolytic bonds that join atoms of different species. Quantum mechanics tells us that only heterolytic bonds are infrared active. Or more simply put, bonds between unlike atoms are capable of absorbing infrared light. Therefore, CO_2 will absorb some wavelengths of infrared light while N_2 and O_2 will not. The ability to absorb infrared light is the molecular property that makes the constituents in the column furthest to the left in Figure 3 greenhouse gases. All are made of different kinds of molecules and, therefore, have heterolytic bonds: all will absorb infrared light.

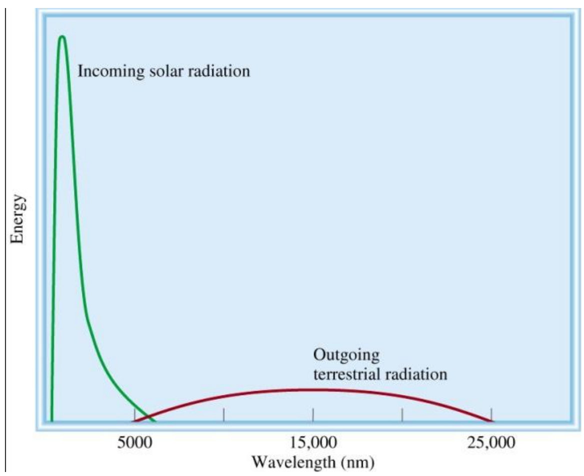

Figure 5. Incoming solar radiation and outgoing terrestrial radiation plotted as a function of energy versus wavelength. Source: *Chemistry*, 9th Edition by Raymond Chang, McGraw Hill, 2007.

But why is this important? To understand the answer, we have to consider the energy balance of our Earth, namely the incoming energy and the outgoing energy. Figure 5 is a gross oversimplification of the energy balance of the earth. It shows the incoming solar radiation (light coming from the sun to Earth) and outgoing terrestrial radiation (light leaving Earth into space). The y-axis represents the energy while the x-axis represents the wavelength of the light in nanometers (nm). What we can see is that the incoming solar radiation is made up of much shorter wavelengths of light than the outgoing terrestrial radiation. The incoming solar radiation is primarily visible light. Infrared light is another form of electromagnetic radiation that has longer wavelength and, therefore, lower energy than visible light. Outgoing terrestrial radiation is infrared light. The energy of the electromagnetic radiation determines how molecules interact with waves. For the most part, visible light passes through atmospheric gases without interacting with the molecules. This is why air is basically transparent to the visible light our eyes can detect. Our atmosphere allows visible incoming solar radiation to come in without being greatly altered. Infrared light, however, will interact with molecules possessing heterolytic bonds. Depending on the specific bond, certain wavelengths will be absorbed and cause the atoms in the bond to vibrate. The absorption of infrared light by CO_2 is demonstrated in Figure 6. This infrared spectrum shows the amount of absorbance per wavelength when infrared light is passed through a sample of CO_2 gas. The x-axis represents wavelength in micrometers (μm), where 1 μm = 1000 nm. We can see that there is a broad absorption peak centered at approximately 15 μm or 15000 nm. If we compare this

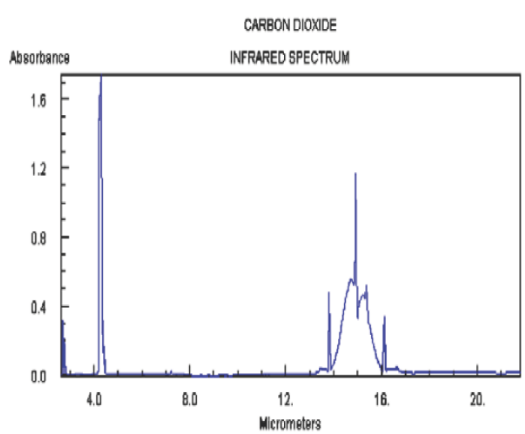

Figure 6. Infrared spectrum of carbon dioxide (CO_2) plotted as absorbance versus wavelength. Where there are peaks in the spectrum, the gas is absorbing the energy of the light. Source: Data compilation copyright by the U.S. Secretary of Commerce on behalf of the U.S.A. Data compiled by: Coblentz Society, Inc.

to the spectrum of outgoing terrestrial radiation in Figure 5, we see that this absorption band coincides with the peak of the outgoing terrestrial radiation. This means that CO_2 in the atmosphere can absorb outgoing terrestrial radiation, thereby trapping that energy in our atmosphere. This warms the atmosphere and the planet. In fact, if there was no naturally occurring CO_2 or other greenhouse gases in our atmosphere, the planet would be approximately 30º C cooler than it is today. This explains why we use the term "greenhouse" for these gases. They allow the incoming visible light energy in but prevent the outgoing terrestrial light energy from escaping, just as the glass in a typical greenhouse does.

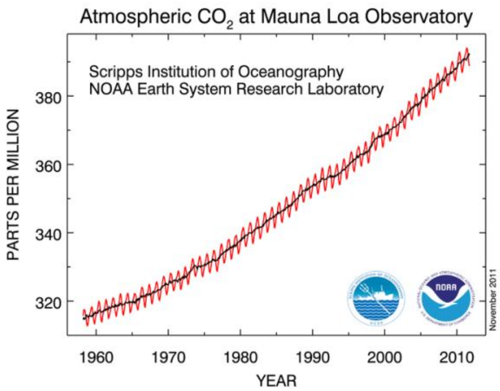

Figure 7. CO_2 concentration data from Mauna Loa Observatory from March 1958 to November 2011. Data is updated weekly and can be found on the National Oceanic and Atmospheric Administration's Earth Systems Research Laboratory site. Source: NOAA ERSL.

A problem arises, however, because the amount of CO_2 in the atmosphere is increasing. And as the amount of CO_2 increases, the amount of outgoing energy being trapped increases and, therefore, the temperature increases. Ice core data indicates that CO_2 concentration was approximately 270 parts per million (or 0.027%) in 1800 before the Industrial Revolution. Figure 7 shows CO_2 concentration data taken at the Mauna Loa Observatory in Hawaii since March 1958. What we can see is the yearly periodic oscillation due to natural growing cycles of plants that absorb CO_2 overlying the overall upward trend in the data that indicates an increase in the amount of CO_2 globally. Most scientists agree that this increase is directly related to the increase in mean surface temperature and global sea level depicted by the data in Figure 1. To understand how humans are responsible for producing CO_2 and the subsequent temperature change, we need to understand how we get energy to drive power our technology, to drive our cars, and to fuel our bodies.

Energy Consumption & CO_2 Emissions

We can think about energy as the energy stored in chemical bonds in the foods we eat or in the fuels we burn to generate heat. Both of these processes are fundamentally the same. They can be considered combustion reactions. Combustion is the reaction that takes place when organic molecules are burned in the presence of oxygen. Organic simply means that the molecule is made primarily of carbon and hydrogen. Organic materials are produced by plants in the process called photosynthesis, which takes energy from the sun and stores it in the chemical bonds of the molecules. Plants use photosynthesis to store energy. Animals take advantage of this process and eat plants for the stored energy. A series of chemical reactions called respiration details how the animals break down the organic molecules to release the stored energy. The end products of combustion reactions and respiration reactions are carbon dioxide (CO_2) and water (H_2O) and energy. Fossil fuel is decomposed plant and animal matter that has been buried and processed by heat and pressure in the earth's mantle for millions of years. It comes in the forms of coal, oil, and natural gas. All the organic material we use for energy from fire wood to food to gasoline (a by-product of oil) derives its energy from the sun through photosynthesis. The chemicals are fundamentally the same.

Using methane (CH_4) as an example, because it is the simplest hydrocarbon and commonly referred to as "natural gas", we can write the chemical reaction

$$CH_4 + 2\,O_2 \rightarrow CO_2 + 2\,H_2O + \text{energy (890 kJ/mol)}.$$

The chemical equation indicates that for every mole of CH_4 burned, one mole of CO_2 and 890 kilojoules (kJ) of energy are produced. [Note that a mole is the fundamental scientific unit for an amount of material and is simply the chemists' way to account for a lot of molecules in a tidy way, where there are 6.022×10^{23} molecules per mole.]

Another common example of a combustion reaction is that of gasoline. Using octane (C_8H_{18}) to represent gasoline, the reaction is shown below:

$$2\ C_8H_{18} + 25\ O_2 \rightarrow 16\ CO_2 + 18\ H_2O + \text{energy (5471 kJ/mol)}.$$

Here, for every 2 moles of C_8H_{18} burned, 16 moles of CO_2 and 5471 kilojoules of energy are produced. Consequently, gasoline is a much more efficient energy source than methane. However, gasoline combustion emits far more CO_2 than methane.

As an example of how we get energy through eating food, glucose ($C_6H_{12}O_6$), a sugar molecule, is broken down through a chemical process called respiration. The overall chemical reaction is

$$C_6H_{12}O_6 + 6\ O_2 \rightarrow 6\ CO_2 + 6\ H_2O + \text{energy (2808 kJ/mol)}.$$

Here, every mole of glucose broken down through respiration yields 6 moles of CO_2 and 2808 kilojoules of energy.

The reactions above are very effective at releasing the energy stored in the reactant molecules. Our bodies, our technologies, and our societies have developed to take advantage of these chemical reactions to produce energy. In this module, we will be focusing on the carbon dioxide that is necessarily produced when we generate our energy through these chemical reactions.

Often when we talk about energy, we are actually talking about electricity. Electricity is the flow of electrical charge and is a secondary source of energy. The primary source of this energy is typically an electromechanical generator, most often driven by steam. While the steam is most often produced by burning fossil fuels, nuclear reactions are also used to generate heat to produce steam to drive turbines and generate electricity. Wind and water (hydroelectric) turbines are also used to produce electricity without steam. Photovoltaic (solar) cells try to replicate photosynthesis and turn the Sun's energy chemical energy that can be used to generate electricity. Burning oil, gas, and coal (fossil fuels) produces carbon dioxide in a similar fashion to burning methane, glucose, and octane. Nuclear, wind, hydroelectric, and solar sources produce electricity without producing carbon dioxide as a by-product.

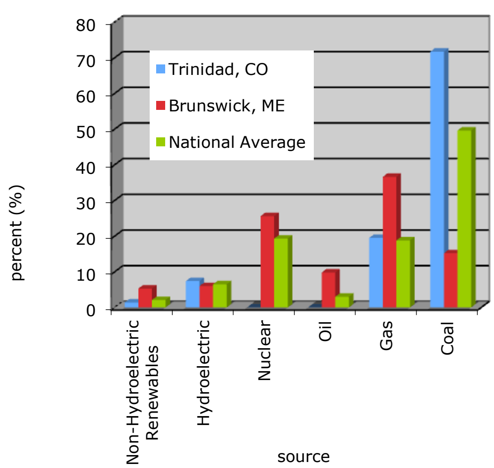

Figure 8. Source of electricity for Trinidad, Colorado, and Brunswick, Maine compared to the national average. Data compiled from http://www.epa.gov/cleanenergy/energy-and-you/how-clean.html, a website that provides information on CO2 production by zip code.

The electricity we consume often comes from a variety of different primary energy sources depending on where we are located and the energy sources available to our local power companies. We can use Figure 8 to compare the electricity sources that involve combustion and, therefore, produce CO_2 (oil, gas, and coal) with the ones that do not (non-hydroelectric, hydroelectric, and nuclear). We can see that the fuel mix for Trinidad, Colorado, uses more combustion sources and far less non-combustion sources than the national average. The fuel mix used in Brunswick, Maine, has a higher

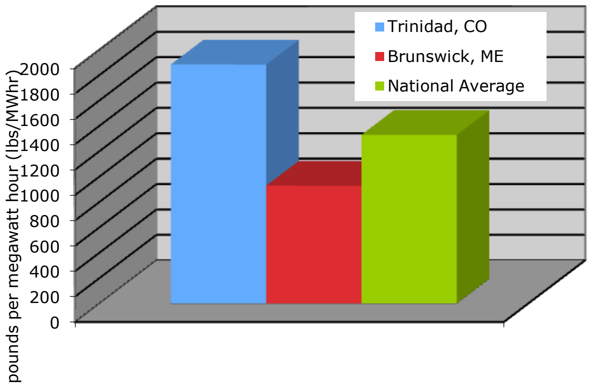

Figure 9. Amount of CO_2 produced in pounds per megaWatt hour for Trinidad, Colorado, and Brunswick, Maine, compared to the national average. Data compiled from http://www.epa.gov/cleanenergy/energy-and-you/how-clean.html, a website that provides information on CO2 production by zip code.

percentage of non-combustions sources than the national average and Trinidad, Colorado. You can compare this to the fuel mix for the electricity you consume by entering your zip code into the website http://www.epa.gov/cleanenergy/energy-and-you/how-clean.html. We can also find the data for the carbon dioxide emissions for each region as depicted in Figure 9. This graph shows how many pounds of CO_2 are produced for each megaWatt hour of electricity that is produced. As we would expect, since Brunswick, Maine, uses fewer combustion sources to produce electricity, less CO_2 is produced there than in Trinidad, Colorado, or nationally. This figure illustrates how humans can make choices that affect the amount of CO_2 emitted through energy production. By choosing energy sources that that do not depend on combustion of fossil fuels, humans can reduce their carbon emissions. We will use the information from Figure 9 to understand how we can make individual choices that affect our personal carbon emissions.

We can use the information in Figure 9 to calculate the amount of carbon dioxide produced for electricity consuming activities. As an example, let's consider burning a 60-Watt (W) light bulb in a desk lamp for 24 hours in Trinidad, Colorado, where 1883 pounds (lbs) of CO_2 is produced for every megawatt hour (MW·hr) of electrical energy used. To do the calculation:

$$60 \text{ W} \times 24 \text{ hr} \times \frac{1883 \text{ lbs } CO_2}{1 \text{ MW} \cdot \text{hr}} \times \frac{1 \text{ MW}}{1 \times 10^6 \text{ W}} = 2.7 \text{ lbs } CO_2$$

Because we have units of *W* and of *mW* and of *hr* in both the numerator and denominator of our fractions, our units cancel to leave us with pounds of carbon dioxide, which is precisely what we were trying to calculate. According to our calculation, 2.7 lbs of CO_2 will be emitted from burning this 60-Watt incandescent light bulb for 24 hours in Trinidad, Colorado. This number is commonly referred to as a carbon footprint.

Since we are talking about a gas, makes more sense to think about this in terms of volume. In other words, how much space would this CO_2 occupy? To estimate the volume of CO_2 in liters (L), we need to know the molecular weight of carbon dioxide (44 grams/mole (g/mol)), the volume of a mole of gas under normal conditions (22.4 liters (L)), and how to convert from English weight in pounds to metric weight in kilograms (kg). (1 kg is equivalent to 2.2 lbs). To estimate the volume of CO_2, our calculation becomes the following:

$$60 \text{ W} \times 24 \text{ hr} \times \frac{1883 \text{ lbs } CO_2}{1 \text{ MW} \cdot \text{hr}} \times \frac{1 \text{ MW}}{1 \times 10^6 \text{ W}} \times \frac{1 \text{ kg}}{2.2 \text{ lb}} \times \frac{1000 \text{ g}}{1 \text{ kg}} \times \frac{1 \text{ mol}}{44 \text{ g}} \times \frac{22.4 \text{ L}}{1 \text{ mol}} = 627 \text{ L}$$

Again, we see that units common to the numerators and denominators cancel out to give us a final answer in liters. Therefore, the amount of space occupied by the CO_2 emissions resulting from burning a 60-Watt (W) light bulb in a desk lamp for 24 hours in Trinidad, Colorado is 627 L. To assist us in visualizing how much volume this actually is, we will relate it to the volume of an everyday object, like a soda can. A soda can contains 12-fluid ounces of liquid. The question becomes how many soda cans is equivalent to 627 L. To estimate the number of soda cans, we have to know that there are 32 fluid ounces in a quart, and we need to know how to convert from the metric to the English system in volume, specifically knowing that 1.06 quarts is equivalent to 1 liter. The calculation is shown below:

$$627 \text{ L} \times \frac{1.06 \text{ quart}}{1 \text{ L}} \times \frac{32 \text{ fluid ounces}}{1 \text{ quart}} \times \frac{1 \text{ can}}{12 \text{ fluid ounces}} = 1{,}772 \text{ cans}$$

This number may impress you, but we can do one more simple calculation to further visualize this volume. Imagine we now stack and align the cans so that we make a solid rectangle with the same number of cans on each side. To figure out how many cans we would need on each side, recall that the volume of such a box is given by $V = x^3$ where x is the side length in number of cans. In our particular case, we have $x^3 = 1772$. To solve for the side length x, we calculate the cube root of 1772, which is 12.1. If we round that down to 12, then we can imagine a stack of soda cans that is 12 cans high by 12 cans deep by 12 cans wide. Given that the dimensions of a soda can are 4.8" high with a 2.5" diameter, this stack of soda cans would be approximately 4.8 feet tall and 2.5 feet deep and wide.

This demonstration illustrates how to translate an abstract idea like the amount of carbon dioxide produced during an energy consuming activity into a tangible object that you can understand better. You should be starting to understand the idea of carbon footprints, the amount of carbon emissions associated with an energy consuming activity. In homework assignment I, you will perform similar calculations to understand how your choice of light bulb affects the amount of CO_2 you are responsible for emitting.

Homework Assignment I

1. The expression below shows the units of four quantities multiplied together. Determine the units of the resulting quantity.

$$\frac{\text{meter}}{\text{second}} \times \frac{\text{miles}}{\text{meter}} \times \frac{\text{second}}{\text{minute}} \times \frac{\text{minute}}{\text{hour}} =$$

2. The equation below shows only the units of each quantity involved. Determine the units for the missing quantity.

$$\frac{\text{acres}}{\text{person} \cdot \text{year}} = \frac{\text{acres}}{\text{ton}} \times \frac{\qquad}{\qquad} \times \frac{\text{pounds}}{\text{person} \cdot \text{day}} \times \frac{\text{days}}{\text{year}}$$

3. Perform the calculation for liters of CO_2 emitted when using a 60-Watt incandescent bulb with your calculator. Do you arrive at the same answer?

We have choices when it comes to picking light bulbs. A 60-Watt incandescent bulb emits the same amount of light as a 13-Watt compact fluorescent bulb and a 3-Watt light emitting diode (LED) bulb. We will now do calculations to determine the volumes of CO_2 emitted when using these alternative bulbs.

4. Modify your calculation in exercise 3 to calculate the carbon footprint of burning a 13-Watt compact fluorescent bulb for 24 hours in Trinidad, Colorado.

5. Can you predict the carbon footprint of using the 3-Watt LED bulb for the same amount of time without actually redoing the whole calculation?

6. Suppose that you operate your lamp 8 hours a day, 5 days a week, for a year. How will the resulting carbon footprints for the three types of bulbs compare?

7. Now suppose that everyone in the United States operated the same lamp 5 days a week for a year. What would the total carbon emissions be for the three different types of bulbs?

8. Challenge yourself! Use your answer from problem 7 and come up with a comparison of the volumes you calculated to an everyday object, like the volume of the Superdome or the cabin of a 737 or an Olympic swimming pool.

In-class Activity: Calculating Carbon Footprints of Transportation

1.) What is the carbon footprint of a 150 lb woman walking 1 mile at a rate of 3 miles per hour, given that she will burn 100 Calories while walking for 20 minutes and that 6 moles of CO_2 is produced for every 2808 kJ burned?

2.) What is the carbon footprint of a 150 lb woman riding a bike at 12 miles per hour, given that she will burn 200 Calories in 20 minutes and that 6 moles of CO_2 is produced for every 2808 kJ burned?

3.) The average gasoline burning car produces 19.4 lbs of CO_2 per gallon of gasoline burned. What is the carbon footprint of driving such a car 1 mile? State your assumptions.

4.) The average diesel burning car produces 10.1 kg of CO_2 per gallon of fuel consumed. What is the carbon footprint of driving such a car 1 mile? State your assumptions.

5.) The Chevy Volt is reported to use 25 kW·hr of electricity to drive 100 city miles. What is the carbon footprint for driving this electric car 1 mile?

6.) A 56-person bus produces 3500 grams of CO_2 per mile. What is the carbon footprint of driving this bus one mile? What is your personal carbon footprint for riding the bus one mile if the bus is half full?

Homework Assignment II

1.) (Carbon Footprint of Water Bottles)
In this problem you are going to estimate the volume of CO_2 produced in the transportation of one 1-liter Aquafina water bottle from Wichita, Kansas, to your doorstep. You will need to use the information given in your pre-class and in-class problems to solve this problem. Assume that transporting 1 metric ton of material one kilometer will produce 102 grams of CO_2 and know that the density of water is one gram per milliliter. State your assumptions and discuss your final answer in comparison with an everyday object.

2.) (Relating Carbon Footprints to Land Area)
Recall from your readings that when fossil fuels are burned, carbon dioxide is emitted into the atmosphere. Some of the carbon dioxide emitted is sequestered, or put more simply absorbed, by the planet's oceans and vegetation, especially forests. Carbon dioxide that is not sequestered remains in the atmosphere. In light of increasing concerns about the warming of the planet due to increasing carbon emissions, scientists have been extensively studying the capacity of the Earth's forests to sequester carbon dioxide. The IPCC Report, ``Land Use, Land-Use Change and Forestry" summarizes carbon sequestration rates obtained as follows: an acre of boreal forest (found in Canada, Alaska, Northern Europe, and Siberia) absorbs an average of 1.3 tons of carbon dioxide annually, while an acre of forest located in a tropical region can absorb an average of 9.8 tons annually. Temperate deciduous forests, common to much of the contiguous United States, absorb an average of 4.9 tons per acre annually. For reference, one acre is approximately the size of an American football field, excluding the end zones.

In this exercise, we explore the capacity of the Earth's forests to absorb the carbon dioxide emissions that we create in our day-to-day lives. In particular, you will estimate the area of forested land required to absorb the carbon dioxide emitted into the atmosphere from driving personal automobiles.

(a.) Using information provided in the in-class problems, estimate the forested land area required annually to sequester your personal carbon dioxide emissions assuming that you drive either a gasoline-powered or diesel-powered car. Note that you will need to take into consideration your own particular driving patterns, your own fuel efficiency, the number of people typically riding in your car, and the geographic region in which you live. (Use the descriptions of boreal, temperate, and tropic given above and choose the one that is most appropriate for your geographic region.) If you do not personally drive such a car, then find someone who does and estimate their land area.

(b.) Estimate the forested land area required annually to sequester your car's CO_2 emissions assuming that you drive the electric-powered Chevy Volt. Use the same assumptions you used about driving habits that you did in part (a).

For the next two exercises, you will apply your understanding of units to contexts different from carbon footprints that, nevertheless, give you a chance to think about consumption choices and sustainability issues.

3.) (Cardboard consumption)
If you took all of the cardboard pizza boxes used in the U.S. over one year and stacked them one on top of the other, about how many miles would this stack reach? (Note: There are 5280 feet in a mile.)

(a.) In order to estimate the height of the stack, what kind of information do you think will be necessary to know or assume?

(b.) Using the 4-step approach discussed in class, estimate the height of the stack. If you consult any outside resources in making your assumptions, please cite them in step 2.

(c.) Mount Everest is 5.5 miles high. How many times larger is the pizza box stack than Mount Everest.

4.) (Water Consumption)
Approximately, how many bath tubs of water does an average person living in the U.S. drink in his/her lifetime?

(a.) In order to estimate the bathtubs of water, what information do you think will be necessary to know or assume?

(b.) Using the four-step approach discussed in class, estimate the number of bathtubs. If you consult any outside resources in making your assumptions, make sure to cite them in step 2.

[Source of problem 4: Ridgway, J., Swan, M., and Burkhardt, H. (2001). Assessing Mathematical Thinking Via FLAG. In: D. Holton and M. Niss (eds.): Teaching and Learning Mathematics at University Level - An ICMI Study. Dordrecht: Kluwer Academic Publishers. pp. 423-430. FLAG Materials accessible at http://www.flaguide.org/. Used with permission.]

Instructor Materials

The following notes are designed to assist instructors in teaching unit conversion and plausible estimation. Alternatively, they can also be used as additional out-of-class reading.

Part one: Introduction to Working with Units

What are dimensions and what are units?

A dimension is a measure of a physical variable. A unit is a way of assigning a numerical value to that dimension. Listed below are seven mutually independent dimensions arising in nature and the units used to represent them. Note that scientists have agreed to an International System of Units (also called *Système international d'unités* and, therefore, abbreviated SI) in order to make scientific writing more uniform.

dimension	SI unit	other common units
length	meter (m)	inch, foot, mile, kilometer
mass	kilogram (kg)	pound, ounce
time	second (s)	minute, year, light year
electric current	ampere (A)	
temperature	Kelvin (K)	Celsius, Fahrenheit
amount of substance	mole (mol)	dozen, gross
luminous intensity	candela (cd)	

All other dimensions are derived and defined in terms of these seven base dimensions using a system of equations. For instance, we know that for speed of an object, we divide the distance the object has traveled by the time it takes it to travel, or in SI units meters/seconds, also written meters per second. Below is a table of a sampling of derived dimensions and their units.

derived dimension	equation	SI units	other common units
Area	length x length	m^2	acre, square mile
Volume	$(\text{length})^3$	m^3	liter, gallon, cup, cubic foot
Speed	length/time	m/s	minute, year, light year
mass density	mass/volume	kg/m^3	gram per milliliter
number density	amount/volume	mol/m^3	mole per liter
Energy	$(\text{length})^2$ x mass/$(\text{time})^2$	$m^2 \cdot kg/s^2$	Joule (J), nutritional Calorie, kiloWatt·hour

While it is easy to see how some of these dimensions are derived, others take a greater understanding of physics to understand their origins. Nevertheless, we can use them to help us set up equations and solve problems.

Understanding units requires that we consider the language of units. The language of prefixes comes from the metric system, where “mega” means 1,000,000 or 10^6, “kilo” means 1000, “hecto” means 100, “centi” means $1/100^{th}$ or 0.01, and “milli” means $1/1000^{th}$. Therefore, a kilometer equals 1000 meters and a megaWatt equals 1,000,000 Watts. It takes 1000 milliliters (mL) to make a liter. And there are 1×10^9 nanometers in a meter. Below is a table with some of the metric prefixes.

Factor	Name	Symbol
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	K
10^{2}	hecto	H
10^{1}	deka	da
10^{-1}	deci	D
10^{-2}	centi	C
10^{-3}	milli	M
10^{-6}	micro	µ
10^{-9}	nano	N

Another point to note about the language of units is how we express fractions. For example, the equation to derive the dimension of speed shows us that we divide the length an object moves by the time it takes the object to move that length, or length/time. We express this verbally as length per time, as in "meters per second" or "miles per hour". Understanding how to translate from the verbal language of problems to the mathematical language of equations makes it necessary to understand that this "per" signifies a fraction and that to say that there are 22.4 liters per mole of gas under standard conditions means that we can write the fraction $\frac{22.4\text{ L}}{1\text{ mol}}$. If this is true then the reciprocal will also be true: that there is one mole of gas in 22.4 L or $\frac{1\text{ mol}}{22.4\text{ L}}$.

We can use these relationships to help solve problems like the ones proposed in the reading assignment and homework assignment I. We will now walk through a simple example to demonstrate how this works. Suppose a person is running at an average speed of 6 miles per hour and the person runs for 3 hours. If you wanted to know how far the person has run, the answer of 18 miles is easy to obtain without thinking deeply. However, this example will help us understand how the units cancel.

First of all, note that 6 miles per hour can be written as $\frac{6\text{ miles}}{1\text{ hour}}$. Also 3 hours can be written as $3\text{ hours}=\frac{3\cdot(1\text{ hour})}{1}$. Now, let's see what happens in close detail when we multiply the 6 miles per hour and the 3 hours.

$$\begin{aligned}\frac{6\text{ miles}}{1\text{ hour}}\times 3\text{ hours} &= \frac{6\text{ miles}}{1\text{ hour}}\times\frac{3\cdot(1\text{ hour})}{1}\\ &= \frac{(6\text{ miles})\cdot 3}{1}\times\frac{1\text{ hour}}{1\text{ hour}}\\ &= (18\text{ miles})\times 1\\ &= 18\text{ miles}\end{aligned}$$

Notice that the common factor of "1 hour" in both the numerator and denominator can be isolated into a separate fraction to equal 1. In other words, the "hours" are divided out from the calculation.

In general, units that appear in both the numerator and denominator will divide out. We can extend our example to demonstrate how we do not have to be as explicit as the previous example when we are doing calculations. Our runner has run 18 miles and want to know how long this distance is in kilometers. One way we can do this is to use the fact that there are 1.6 kilometers in one mile or $\frac{1.6 \text{ km}}{1 \text{ mile}}$. If we want to convert 18 miles to kilometers then we want to multiply 18 miles by this conversion factor so that the mile units cancel out and we are left with an answer in kilometers

$$18 \text{ miles} \times \frac{1.6 \text{ km}}{1 \text{ mile}} = 28.8 \text{ km}.$$

Typically, our problems are not this simple and it requires multiple conversion factors to arrive at the correct answer. Let's look at an example where we multiply by two ratios to convert from one unit to another. For example, suppose we measured a distance of 250 feet and wanted to know how long this distance is in kilometers. If we start with a measure in feet, we can use the fact that there are 5280 feet in a 1 mile or $\frac{5280 \text{ ft}}{1 \text{ mile}}$. We can use this conversion factor along with the ratios we wrote above which relate kilometers to miles. The key here is to create a chain of ratios so that appropriate units cancel and so that the desired units end up in the numerator. To convert 250 feet to kilometers, we use the following chain of ratios

$$250 \text{ feet} \times \frac{1 \text{ mile}}{5280 \text{ feet}} \times \frac{1.6 \text{ km}}{1 \text{ mile}} = 7.58 \times 10^{-2} \text{ km}$$

Note that we had to use the reciprocal of our "miles to feet" ratio as it was presented in order to obtain the correct final answer.

Another important point to make is that you can make up a conversion factor like the ones above with any equality, even if the units seem to be strange or unscientific. Returning to our runner, lets calculate how many marathons she has run (or what fraction thereof) during her 3 hours. To do this, we need to know that a marathon is 42.195 km (or 26 miles and 385 yards) or $\frac{1 \text{ marathon}}{42.195 \text{ km}}$. Putting the whole problem together

$$3 \text{ hours} \times \frac{6 \text{ miles}}{1 \text{ hour}} \times \frac{1.6 \text{ km}}{1 \text{ mile}} \times \frac{1 \text{ marathon}}{42.195 \text{ km}} = 0.68 \text{ marathons}$$

Our runner has run just over two-thirds of a marathon in 3 hours. If we want to figure out how long it would take her to run a full marathon at the same average speed, we can create a new conversion factor from our calculation by saying it takes 3 hours per 0.68 marathons; therefore,

$$1 \text{ marathon} \times \frac{3 \text{ hours}}{0.68 \text{ marathon}} = 4.4 \text{ hours}.$$

Another way to tackle this same problem is to recognize that we created a linear relationship between time running and marathons completed such that we can calculate the answer for how long a full marathon (or even twelve marathons) would take to complete by simply setting up two ratios that are equal to each other with our unknown quantity represented by *x* and solve for *x*. Explicitly,

$$\frac{1 \text{ marathon}}{x \text{ hours}} = \frac{0.68 \text{ marathon}}{3 \text{ hours}}$$

Now, simply cross-multiply and divide to get

$$1 \text{ marathon} \times 3 \text{ hours} = 0.68 \text{ marathon} \times x \text{ hours}$$

Therefore,

$$\frac{1 \text{ marathon} \times 3 \text{ hours}}{0.68 \text{ marathon}} = x \text{ hours}$$

Or again 4.4 hours.

Finally, note that when we state our answers we always state the units. When working with dimensions, unit-less values are typically meaningless. Sometimes the units are implied, but in order to give complete answers, the units should always be included.

Part two: Introduction to Plausible Estimation

We have just seen how using units can help us set up equations to answer questions. Another tool that allows us to answer questions is making plausible estimations by using reasonable assumptions. We now will go through a strategy to break down the estimation process in four steps. These steps are

- **I.)** Determine the units of the quantity that you want to estimate. Write the units in a fraction form when appropriate.
- **II.)** List all the needed information to write the equation, written in fraction form when appropriate.
- **III.)** Combine the information from step **II** using the concept of unit conversion to obtain the units of the quantity you want to estimate from step **I**. Perform the numerical calculation and simplify the units by canceling.
- **IV.)** Using complete sentences, summarize your findings in the context of the question. Make sure to state all assumptions.

To demonstrate this procedure, we revisit the carbon footprint calculation from the pre-class reading as our example. Suppose we need to estimate the CO_2 emissions for using a lamp for a certain length of time.

- **I.)** Determine the units of the quantity that you want to estimate. Write the units in a fraction form when appropriate.

  This can also be considered "What you want to know". Figuring out what the question is asking is often the hardest part of the problem. The question is asking how much carbon dioxide. Since carbon dioxide is a gas, expressing the answer as a volume makes sense. While m^3 is the SI unit for volumes, we will pick a unit that is more familiar to us, namely liters (L).

  L of CO_2

II.) List all the needed information to write the equation, written in fraction form when appropriate.

This can also be considered "what you know". It is helpful to state what assumptions you are making here so that you do not forget them in step ***IV****. From the reading know the following facts*

60-Watt bulb (assuming we are using the same bulb as the example in the text)
24 hours
$\frac{1883 \text{ lbs } CO_2}{1 \text{ MW·hr}}$ (assuming we are in Trinidad, Colorado)

Some of what you are told in the reading is independent of the specific problem. These are things you know or assume in order to answer the question. For instance, we know we want a volume of CO_2 and we are given a mass of CO_2. To get from mass of CO_2 to volume of CO_2 we need to know the amount of CO_2, or the number of moles of CO_2. We must find a way to get from one dimension to the other by using a conversion factor or a series of conversion factors. These are given in the text but could also be looked up if you did not know these values. The following ratios will help us set up the equation.

$\frac{1 \text{ mol } CO_2}{44 \text{ grams}}$

$\frac{22.4 \text{ L } CO_2}{1 \text{ mol } CO_2}$ (Assuming room temperature and atmospheric pressure)

$\frac{1 \text{ kg}}{2.2 \text{ lbs}}$

$\frac{1000 \text{ g}}{1 \text{ kg}}$

$\frac{1 \text{ MW}}{1 \times 10^6 \text{ W}}$

III.) Combine the information from step **II** using the concept of unit conversion to obtain the units of the quantity you want to estimate from step **I**. Perform the numerical calculation and simplify the units by canceling.

Now we set up the equation that uses our known information (from step ***II****) to estimate our desired quantity (step* ***I****). You will find it helpful if you start with the quantity in step II that has the units you desire in your answer. In this case, we start with* $\frac{22.4 \text{ L } CO_2}{1 \text{ mol } CO_2}$ *. We multiply this quantity by a quantity that will cancel out the moles, namely* $\frac{1 \text{ mol } CO_2}{44 \text{ grams}}$*. Now we multiply by a quantity that will cancel out the grams,* *namely* $\frac{1000\text{ g}}{1\text{ kg}}$. *We continue in this way until all unwanted units are canceled, leaving us with* L CO_2. *We set this up as follows:*

$$\frac{1883\text{ lbs }CO_2}{1\text{ MW}\cdot\text{hr}}\times\frac{1\text{ MW}}{1\times10^6\text{ W}}\times\frac{1\text{ kg}}{2.2\text{ lb}}\times\frac{1000\text{ g}}{1\text{ kg}}\times\frac{1\text{ mol}}{44\text{ g}}\times\frac{22.4\text{ L}}{1\text{ mol}}\times 60\text{ W}\times 24\text{ hours}\times = 627\text{ L}$$

IV.) Using complete sentences, summarize your findings in the context of the question. Make sure to state all assumptions.

According to our calculation, 627 liters of CO_2 will be emitted from using a 60-Watt light bulb for 24 hours, assuming that the carbon dioxide is at room temperature and atmospheric pressure and that our light bulb is in Trinidad, Colorado. Under different atmospheric conditions, the volume would be different. Also, if we change locations to one where the electric company is using more non-combustible energy sources, then the value would be less.

Following this strategy, we can come up with equations to lead us to answers for questions that seem farfetched. These "back-of-the-envelope" or "bar napkin" calculations can be very helpful in solving interesting questions. There are times when it is necessary to make reasonable assumptions, when you do not actually know all the information to get an answer. What if we wanted to know the total CO_2 emissions if all Americans used a 60-watt bulb for their reading light by their bedside. We will follow the strategy, using our estimate from the previous answer as a new conversion factor.

I.) L of CO_2

II.) *We just derived a new conversion factor that we can now use namely*

$$\frac{627\text{ L }CO_2}{24\text{ hours of light bulb use}}$$

Now we have to make some assumptions and maybe look up some statistics

313,000,000 people in the United States (*population data on-line*)

60% of Americans have a bedside lamp (*assumption that the population is partially comprised of non-readers, including children too young to read*), or, put differently

$$\frac{60\text{ readers}}{100\text{ total people}}$$

$\frac{30\text{ minutes of lamp use}}{1\text{ day}}$ (*assumption based on personal habits*)

$$\frac{1\text{ hour}}{60\text{ minutes}}$$

III.) We are targeting liters of CO_2. Therefore, we begin with a conversion factor from step 2 that has liters in it. Here, we start with $\frac{630\text{ L}}{24\text{ hours}}$. The rest of the equation is then completely determined.

$$\frac{627\text{ L}}{24\text{ hours}} \times \frac{60\text{ people with lamps}}{100\text{ total people}} \times \frac{30\text{ minutes}}{1\text{ day}} \times \frac{1\text{ hour}}{60\text{ minutes}} \times 3.13 \times 10^{8}\text{ people} = \frac{2.5\ \times 10^{9}\text{L}}{\text{day}}$$

IV.) Assuming that only 60% of the American population uses a bedside lamp and that they use it for approximately the same amount of time that I use mine, namely 30 minutes, then the carbon emissions for this energy consuming activity is 2.5 x 10^9 liters every day. This is 2,500,000,000 or two and a half billion liters per day!

Clearly, the estimation is only as good as our assumptions. Assumptions that greatly over-predict or greatly under-predict a value are useful to establish upper and lower limits to the answer to a question.

Homework Assignment I Solutions (answers in red)

1. The expression below shows the units of four quantities multiplied together. Determine the units of the resulting quantity.

$$\frac{\text{meter}}{\text{second}} \times \frac{\text{miles}}{\text{meter}} \times \frac{\text{second}}{\text{minute}} \times \frac{\text{minute}}{\text{hour}} = \frac{\text{miles}}{\text{hour}}$$

2. The equation below shows only the units of each quantity involved. Determine the units for the missing quantity.

$$\frac{\text{acres}}{\text{person} \cdot \text{year}} = \frac{\text{acres}}{\text{ton}} \times \frac{\text{ton}}{\text{pounds}} \times \frac{\text{pounds}}{\text{person} \cdot \text{day}} \times \frac{\text{days}}{\text{year}}$$

3. Perform the calculation for liters of CO_2 emitted when using a 60-Watt incandescent bulb with your calculator. Do you arrive at the same answer?

Answer should be the same. This is simply a check to see that the student can enter the numbers into their calculator correctly.

We have choices when it comes to picking light bulbs. A 60-Watt incandescent bulb emits the same amount of light as a 13-Watt compact fluorescent bulb and a 3-Watt light emitting diode (LED) bulb. We will now do calculations to determine what the volumes of gas produced using these alternative bulbs will be.

4. Modify your calculation in exercise 3 to calculate the carbon footprint of burning a 13-Watt compact fluorescent bulb for 24 hours in Trinidad, Colorado. Interpret your answer in terms of carbon emissions. Use complete sentences.

$$13\ \text{W} \times 24\ \text{hr} \times \frac{1883\ \text{lbs}\ \text{CO}_2}{1\ \text{MW} \cdot \text{hr}} \times \frac{1\ \text{MW}}{1 \times 10^6\ \text{W}} \times \frac{1\ \text{kg}}{2.2\ \text{lb}} \times \frac{1000\ \text{g}}{1\ \text{kg}} \times \frac{1\ \text{mol}}{44\ \text{g}} \times \frac{22.4\ \text{L}}{1\ \text{mol}} = 136\ \text{L}$$

By changing from a 60-Watt incandescent bulb to a 13-Watt compact fluorescent bulb, the emissions for using the light for the same period in time decrease from 630 L to 136 L.

5. Can you predict the carbon footprint of using the 3-Watt LED bulb for the same amount of time without actually redoing the whole calculation?

There are several ways to do this all require that the students realize that there is a linear relationship between the Wattage of the bulb and the CO_2 emissions. One way is to set up a ratio and then solve for the missing term (x) by cross-multiplying then dividing

$$\frac{3\text{ W}}{x\text{ L}} = \frac{60\text{ W}}{627\text{ L}}$$

$x = 31.4$

More advanced students may realize that 3 is 5% of 60; therefore, the emissions from the 3-W bulb should be 5% of that from the 60-W bulb, or 5% of 627 L which is, again 31.4 L.

6. Suppose that you operate your lamp 8 hours a day, 5 days a week, for a year. How will the resulting carbon footprints for the three types of bulbs compare?

First we must determine how many hours we are considering.

$$\frac{8\text{ hr}}{\text{day}} \times \frac{5\text{ days}}{1\text{ week}} \times \frac{52\text{ week}}{1\text{ year}} = 2080\text{ hours}$$

Then we can do the whole calculation again using 2080 hours in place of 24 hours. Or we can see that there is a linear relationship between usage time and CO_2 emissions and set up a ratio like we did in problem 5. Therefore, if 24 hours resulted in 627 L of CO_2 then 2080 hours will produce 54,340 L. We can use answers from problems 4 and 5 to get the other two answers.

136 L * 2080 hours/24 hours = 11,787 L for the compact fluorescent bulb

And 31.4 * 2080 hours/24 hours = 2721 L for the LED bulb

7. Now suppose that everyone in the United States operated the same lamp 5 days a week for a year. Compare the carbon emissions for the three different bulbs. State your assumptions.

Assume that the population of the United States is 313 million people. We have just calculated the carbon footprint for a single person using the three types of bulbs for a year. We simply have to multiply these numbers by 313 million (or 3.13×10^8) to get the answers of 1.7×10^{13} liters (incandescent bulb), 3.7×10^{12} liters (compact fluorescent bulb), 8.5×10^{11} liters (LED bulb). These numbers are orders of magnitude different in size, which is amazing. By changing our light bulbs we can decrease carbon emissions by orders of magnitude.

8. Challenge yourself! Use your answer from problem 7 and come up with a comparison of the volumes you calculated to an everyday object, like the volume of the Superdome or cabin of a 737 or an Olympic swimming pool.

The volume of the Superdome is reported to be 155 million cubic feet

$$\frac{1.55\times10^{8}\ \text{ft}^3}{1\ \text{Superdome}}\times\frac{(12\ \text{in})^3}{(1\ \text{ft})^3}\times\frac{(2.54\ \text{cm})^3}{(1\ \text{in})^3}\times\frac{1\ \text{mL}}{1\ \text{cm}^3}\times\frac{1\ \text{L}}{1000\ \text{mL}}=\frac{4.4\times10^{9}\ \text{L}}{\text{Superdome}}$$

And the volume of gas produce by the incandescent bulb is 1.7×10^{13} L

$$1.7\times10^{13}\ \text{L}\times\frac{1\ \text{Superdome}}{4.4\times10^{9}\ \text{L}}=3.86\times10^{3}\ \text{Superdomes}$$

Or approximately 4000 Superdomes. The compact fluorescent bulb emitting 4.0×10^{12} liters corresponds to approximately 900 Superdomes, and the LED bulb corresponds to approximately 200 Superdomes.

In-class Activity: Calculating Carbon Footprints of Transportation Solutions (answers in red)

1.) What is the carbon footprint of a 150 lb woman walking 1 mile at a rate of 3 miles per hour, given that she will burn 100 Calories while walking for 20 minutes and that 6 moles of CO_2 is produced for every 2808 kJ burned?

$$1\text{ mile}\times\frac{1\text{ hr}}{3\text{ mile}}\times\frac{60\text{ min}}{1\text{ hr}}\times\frac{100\text{ Cal}}{20\text{ min}}\times\frac{1\text{ kcal}}{1\text{ Cal}}\times\frac{1000\text{ cal}}{1\text{ kcal}}\times\frac{4.184\text{ J}}{1\text{ cal}}\times\frac{1\text{ kJ}}{1000\text{ J}}\times\frac{6\text{ mol}}{2808\text{ kJ}}\times\frac{22.4\text{ L}}{1\text{ mol}}=20\text{ L}$$

2.) What is the carbon footprint of a 150 lb woman riding a bike at 12 miles per hour, given that she will burn 200 Calories in 20 minutes and that 6 moles of CO_2 is produced for every 2808 kJ burned?

$$1\text{ mile}\times\frac{1\text{ hr}}{12\text{ mile}}\times\frac{60\text{ min}}{1\text{ hr}}\times\frac{200\text{ Cal}}{20\text{ min}}\times\frac{1\text{ kcal}}{1\text{ Cal}}\times\frac{1000\text{ cal}}{1\text{ kcal}}\times\frac{4.184\text{ J}}{1\text{ cal}}\times\frac{1\text{ kJ}}{1000\text{ J}}\times\frac{6\text{ mol}}{2808\text{ kJ}}\times\frac{22.4\text{ L}}{1\text{ mol}}=10\text{ L}$$

3.) The average gasoline burning car produces 19.4 lbs of CO_2 per gallon of gasoline burned. What is the carbon footprint of driving such a car 1 mile? State your assumptions.

$$1\text{ mile}\times\frac{1\text{ gallon}}{21\text{ mile}}\times\frac{19.4\text{ lb}}{1\text{ gallon}}\times\frac{1\text{ kg}}{2.2\text{ lb}}\times\frac{1000\text{g}}{1\text{ kg}}\times\frac{1\text{ mol}}{44\text{ g}}\times\frac{22.4\text{ L}}{1\text{ mol}}=214\text{ L}$$

Assuming 21 mile/gallon.

4.) The average diesel burning car produces 10.1 kg of CO_2 per gallon of fuel consumed. What is the carbon footprint of driving such a car 1 mile? State your assumptions.

Assuming 40 mile/gallon.

$$1\text{ mile}\times\frac{1\text{ gallon}}{40\text{ mile}}\times\frac{10.1\text{ kg}}{1\text{ gallon}}\times\frac{1000\text{g}}{1\text{ kg}}\times\frac{1\text{ mol}}{44\text{ g}}\times\frac{22.4\text{ L}}{1\text{ mol}}=129\text{ L}$$

5.) The Chevy Volt is reported to use 25 kW·hr of electricity to drive 100 city miles. What is the carbon footprint for driving this electric car 1 mile?

$$1\text{ mile}\times\frac{25\text{ kW·hr}}{100\text{ mile}}\times\frac{1000\text{ W}}{1\text{ kW}}\times\frac{1\text{ MW}}{1\times10^{6}\text{ W}}\times\frac{1883\text{ lb }CO_2}{1\text{ MW·hr}}\times\frac{1\text{ kg}}{2.2\text{ lb}}\times\frac{1000\text{ g}}{1\text{ kg}}\times\frac{1\text{ mol}}{44\text{ g}}\times\frac{22.4\text{ L}}{1\text{ mol}}=109\text{ L}$$

6.) A 56-person bus produces 3500 grams of CO_2 per mile. What is the carbon footprint of driving this bus one mile? What is your personal carbon footprint for riding the bus one mile if the bus is half full?

$$1\ \text{mile} \times \frac{3500\ \text{g}}{1\ \text{mile}} \times \frac{1\ \text{mol}}{44\ \text{g}} \times \frac{22.4\ \text{L}}{1\ \text{mol}} = 1782\ \text{L}$$

1782 L is the total amount of CO_2 emitted by the bus in one mile. To figure out each riders' share, if there are 56/2 = 28 people.

$$\frac{1782\ \text{L}}{1\ \text{bus}} \times \frac{1\ \text{bus}}{28\ \text{people}} = 64\ \text{L}$$

Links used to develop in-class problems:

http://www.healthstatus.com/calculate/cbc

http://www.epa.gov/otaq/climate/documents/420f11041.pdf

www.physics.uci.edu/~silverma/voltmileage.html

http://www.epa.gov/ttn/chief/conference/ei11/mobile/wilson.pdf

Homework Assignment II Solutions (answers in red)

1.) (Carbon Footprint of Water Bottles)
In this problem you are going to estimate the volume of CO_2 produced in the transportation of one 1-liter Aquafina water bottle from Wichita, Kansas, to your doorstep. You will need to use the information given in your pre-class and in-class problems to solve this problem. Assume that transporting 1 metric ton of material one kilometer will produce 102 grams of CO_2 and know that the density of water is one gram per milliliter. State your assumptions and discuss your final answer in comparison with an everyday object.

Brunswick, Maine, is 2822.5 km from Wichita, Kansas according to Google maps. We are assuming that the weight of the actual bottle is negligible.

$$1 \text{ bottle} \times 2822.5 \text{ km} \times \frac{1000 \text{ mL}}{1 \text{ bottle}} \times \frac{1 \text{ g } H_2O}{1 \text{ mL } H_2O} \times \frac{1 \text{ kg } H_2O}{1000 \text{ g } H_2O} \times \frac{1 \text{ metric ton } H_2O}{1000 \text{ kg } H_2O} \times \frac{102 \text{ g } CO_2}{1 \text{ ton } H_2O{\cdot}1 \text{ km}}$$

$$\times \frac{1 \text{ mol } CO_2}{44 \text{ g } CO_2} \times \frac{22.4 \text{ L } CO_2}{1 \text{ mol } CO_2} = 147 \text{ L } CO_2$$

The transportation of a liter of water from Wichita, Kansas, to Brunswick, Maine, results in a carbon emission of 147 L. To put this in perspective, we can compare this volume to the volume 16”-diameter beach ball.

$$\frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times 8^3 \text{ in}^3 \times \frac{2.54^3 \text{cm}^3}{1^3 \text{ in}^3} \times \frac{1 \text{ mL}}{1 \text{ cm}^3} \times \frac{1 \text{ L}}{1000 \text{ mL}} = 35 \text{ L.}$$

The ball will hold 35 L; therefore,

$$147 \text{ L} \times \frac{1 \text{ ball}}{35 \text{ L}} = 4.2 \text{ balls}$$

The volume of gas emitted for transporting a liter of water from Wichita, Kansas, to Brunswick, Maine, is approximately 4 beach balls. The carbon footprint of drinking water out of your tap is basically zero.

2.) (Relating Carbon Footprints to Land Area)
Recall from your readings that when fossil fuels are burned, carbon dioxide is emitted into the atmosphere. Some of the carbon dioxide emitted is sequestered, or put more simply absorbed, by the planet's oceans and vegetation, especially forests. Carbon dioxide that is not sequestered remains in the atmosphere. In light of increasing concerns about the warming of the planet due to increasing carbon emissions, scientists have been extensively studying the capacity of the Earth's

forests to sequester carbon dioxide. The IPCC Report, ``Land Use, Land-Use Change and Forestry" summarizes carbon sequestration rates obtained as follows: an acre of boreal forest (found in Canada, Alaska, Northern Europe, and Siberia) absorbs an average of 1.3 tons of carbon dioxide annually, while an acre of forest located in a tropical region can absorb an average of 9.8 tons annually. Temperate deciduous forests, common to much of the contiguous United States, absorb an average of 4.9 tons per acre annually. For reference, one acre is approximately the size of an American football field, excluding the end zones.

In this exercise, we explore the capacity of the Earth's forests to absorb the carbon dioxide emissions that we create in our day to day lives. In particular, you will estimate the area of forested land required to absorb the carbon dioxide emitted into the atmosphere from driving personal automobiles.

(a.) Using information provided in the in-class problems, estimate the forested land area required annually to sequester your personal carbon dioxide emissions assuming that you drive either a gasoline-powered or diesel-powered car. Note that you will need to take into consideration your own particular driving patterns, your own fuel efficiency, the number of people typically riding in your car, and the geographic region in which you live. (Use the descriptions of boreal, temperate, and tropic given above and choose the one that is most appropriate for your geographic region.) If you don't personally drive such a car, then find someone who does and estimate their land area.

If we assume that a person lives in a region having temperate deciduous forests, has 1.57 people in the car (U.S. average ridership), drives 40 miles a day for 365 days each year, and owns a gasoline-powered car with an average fuel efficiency of 26 miles per gallon, then the estimated per capita sequestration land area is

$$\frac{1 \text{ acre}}{4.9 \text{ tons}} \times \frac{1 \text{ ton}}{2000 \text{ lbs}} \times \frac{19.4 \text{ lbs}}{1 \text{ gallon}} \times \frac{1 \text{ gallon}}{26 \text{ miles}} \times \frac{40 \text{ miles}}{1 \text{ day}} \times \frac{365 \text{ day}}{1 \text{ year}} = 1.1 \text{ acres}$$

Therefore, an estimated 1.1 acres of temperate forest land is required to sequester the carbon dioxide emissions of this one car annually. So, each rider's share of this land area is

$$\frac{1.1 \text{ acre}}{1 \text{ car}} \times \frac{1 \text{ car}}{1.57 \text{ people}} = 0.71 \text{ acres per person}$$

An estimated 0.71 acres of temperate deciduous forest would be required annually to absorb this person's carbon dioxide auto emissions from driving a gasoline-powered car. This is nearly 75% of a football field. Multiply this number by the population of the United State (313 million) and we see that, if we all drive like average Americans, we need 2.3 x 10^8 acres of forested land to sequester the CO_2 from driving cars. This corresponds to about 1/10 of the total land area of the entire United States (2.3 billion acres).

(b.) Estimate the forested land area required annually to sequester your car's CO_2 emissions assuming that you drive the electric-powered Chevy Volt. Use the same assumptions you used about driving habits that you did in part (a).

$$\frac{1\text{ acre}}{4.9\text{ tons}} \times \frac{1\text{ ton}}{2000\text{ lbs}} \times \frac{1883\text{ lbs}}{1\text{ MW}\cdot\text{hr}} \times \frac{1\text{ MW}}{1\times 10^6\text{ W}} \times \frac{1000\text{ W}}{1\text{ kW}} \times \frac{25\text{ kW}\cdot\text{hr}}{100\text{ miles}} \times \frac{40\text{ miles}}{1\text{ day}} \times \frac{365\text{ day}}{1\text{ year}} = 0.70\text{ acres}$$

We calculate an estimated 0.70 acres of temperate forest land is required to sequester the carbon dioxide emissions of this one car annually. Therefore, each rider's share of this land area is

$$\frac{0.70\text{ acre}}{1\text{ car}} \times \frac{1\text{ car}}{1.57\text{ people}} = 0.45\text{ acres per person}$$

an estimated 0.44 acres of temperate deciduous forest would be required annually to absorb this person's carbon dioxide emissions from driving an electric car. This is nearly half of a football field.

For the next two exercises, you will apply your understanding of units to contexts different from carbon footprints that, nevertheless, give you a chance to think about consumption choices and sustainability issues.

3.) (Cardboard consumption)
If you took all of the cardboard pizza boxes used in the U.S. over one year and stacked them one on top of the other, about how many miles would this stack reach? (Note: There are 5280 feet in a mile.)

(a.) In order to estimate the height of the stack, what kind of information do you think will be necessary to know or assume?

Responses to this question will vary with the types of assumptions students make

(b.) Using the 4-step approach discussed in class, estimate the height of the stack. If you consult any outside resources in making your assumptions, please cite them in step 2.

a student assumes that a typical pizza box contains 8 slices of pizza, the average person in the U.S. eats 46 slices a year, there are 313 million people in the U.S., and the height of an average pizza box is 2 inches, then they would arrive at the following estimate:

$$\frac{1\text{ mile}}{5{,}280\text{ feet}} \times \frac{1\text{ foot}}{12\text{ inches}} \times \frac{2\text{ inches}}{1\text{ box}} \times \frac{1\text{ box}}{8\text{ slices}} \times \frac{46\text{ slices}}{1\text{ person}} \times 3.13\times10^8\text{ people} = 56{,}810\text{ miles}$$

(c.) Mount Everest is 5.5 miles high. How many times larger is the pizza box stack than Mount Everest?

According to the estimate from part (b), the stack is 10,329 times the height of Mount Everest!

4.) (Water Consumption) Approximately, how many bath tubs of water does an average person living in the U.S. drink in his/her lifetime?

(a.) In order to estimate the bath tubs of water, what information do you think will be necessary to know?

Responses will vary depending on assumptions students make.

(b.) Using the four-step approach discussed in class, estimate the number of bath tubs. If you consult any outside resources in making your assumptions, make sure to cite them in step 2.

Note: There are several ways that a student might estimate the volume of a bath tub. Some might try to get some direct estimates from the internet. Some might assume that a bath tub is roughly rectangular and estimate the volume from using side length dimensions. And, some might even estimate the volume by first estimating the volumetric flow rate of their tub at home and then multiplying that number by the amount of time required to fill their tub. (The flow rate can be estimated by seeing how long it takes to fill a container of known volume like a 2-liter bottle or a gallon milk bottle.)

If, for example, a student assumes that a tub is roughly rectangular with dimensions 5 feet by 32 inches by 18 inches, then the volume of a bath tub is about 35,000 in^3. Additionally, students will need to make an assumption about the amount a person drinks, say 8 glasses a day. And they will need to make an assumption about the average lifetime of a person in the U.S., say 78 years old. With these assumptions, a student will arrive at the following estimate:

$$\frac{1\text{ tub}}{35{,}000\text{ in}^3} \times \frac{1^3\text{ in}^3}{2.54^3\text{ cm}^3} \times \frac{1\text{ cm}^3}{1\text{ mL}} \times \frac{1000\text{ mL}}{1\text{ L}} \times \frac{1\text{ L}}{1.06\text{ quarts}} \times \frac{1\text{ quart}}{4\text{ cups}} \times \frac{8\text{ cups}}{1\text{ day}} \times \frac{365\text{ days}}{1\text{ year}} \times 78\text{ years} = 94\text{ tubs}$$

Source of problem 4: Ridgway, J., Swan, M., and Burkhardt, H. (2001). Assessing Mathematical Thinking Via FLAG. In: D. Holton and M. Niss (eds.): Teaching and Learning Mathematics at University Level - An ICMI Study. Dordrecht: Kluwer Academic Publishers. pp. 423-430. FLAG Materials accessible at http://www.flaguide.org/. Used with permission.)